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| EGC_Black | **MATHEMATICS: SPECIALIST 3 & 4**  **INVESTIGATION 1** |

**COMPLEX NUMBERS USING *eiθ***

**Time Allowed: 55 minutes Total Marks: 43**

**Course-related information**

The concepts and skills included in this investigation relate to the following dot points within the WA Mathematics Specialist syllabus.

3.1.1 review real and imaginary parts Re() and Im() of a complex number 

3.1.2 review Cartesian form

3.1.3 review complex arithmetic using Cartesian form

3.1.4 use the modulus  of a complex number  and the argument  of a non-zero complex number  and prove basic identities involving modulus and argument

**Background information**

It is assumed that students have completed Methods 1.2.7 – 1.2.8. 1.2.14, 1.2.16 (Trigonometric functions), Specialist 2.1.1, 2.1.3 (The basic trigonometric functions), Specialist 2.3.7 – 2.3.10 (Complex numbers), Specialist 3.1.1 – 3.1.3 (Cartesian forms) and Specialist 3.1.4.

**Task conditions**

For this task the use of the calculator is not permitted.

**Question 1** **(13 marks)**

Given

(a) show that

(i) , (4)

(ii) . (2)

Hence,

(b) determine expressions for and for in terms of and ,

(3)

(c) solve

(i) , (2)

(ii) . (2)

**Question 2**  **(11 marks)**

The complex number can be written in three forms:

Cartesian:

Polar: where and

Exponential where and

(a) Consider the complex number ,

(i) express in exponential form, (2)

(ii) express in exponential form, (2)

(iii) express in Cartesian form. (2)

(b) Express in Cartesian form. (5)

**Question 3**  **(19 marks)**

The hyperbolic trigonometric functions and are defined by

(a) Show that

(i) (2)

(ii) (2)

(iii) (2)

(b) Express in terms of and . (3)

(c) Show that and determine an expression for in terms of .

(4)

(d) Write and in terms of the hyperbolic functions. (3)

(e) Express , in Cartesian form. (3)

**End of questions**

**Complex numbers using *eiθ***

**In-class investigation**

**Solutions and marking key**

**Question 1(a)(i)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Correctly substitutes for * Simplifies the terms * Identifies the expansion for * identifies the expansion for | 1  1  1  1 |

**Question 1(a)(ii)**

|  |  |
| --- | --- |
| Solution | |
| Replace  with , | |
| Mathematical behaviours | Marks |
| * Correctly substitutes for * Simplifies the terms | 1  1 |

**Question 1(b)**

|  |  |
| --- | --- |
| Solution | |
| Given that  and | |
| Mathematical behaviours | Marks |
| * Recognises the need for addition to obtain * Recognises the need for subtraction to obtain * Correct expressions for  and for | 1  1  1 |

**Question 1(c)(i)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Rewrites the equation in terms of * Solves for | 1  1 |

**Question 1(c)(i)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Rewrites the equation in terms of * Solves for | 1  1 |

**Question 2(a)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Correct modulus * Correct argument | 1  1 |

**Question 2(b)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Correct modulus * Correct argument | 1  1 |

**Question 2(c)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Applies definition of * Expresses  in Cartesian form | 1  1 |

**Question 2(d)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Expresses  in exponential form * Expresses  in exponential form * Applies definition of * Correct real component * Correct imaginary component | 1  1  1  1  1 |

**Question 3(a)(i)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Substitutes  for * Establishes | 1  1 |

**Question 3(a)(ii)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Substitutes  for * Establishes | 1  1 |

**Question 3(a)(iii)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Substitutes for  and * Simplifies resulting expression | 1  1 |

**Question 3(b)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Writes expression for * Factorises numerator * Rewrites in terms of and | 1  1  1 |

**Question 3(c)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Substitutes for * Recognises * Substitutes for * Establishes | 1  1  1  1 |

**Question 3(d)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Substitutes  for * Establishes * Establishes | 1  1  1 |

**Question 3(e)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * Substitutes for * Applies trig identity * Simplifies to establish | 1  1  1 |